High energy physics in the duality era

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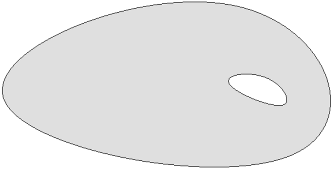
Overview

1. Locality
2. Gauge Symmetry
3. Spacetime Geometry

Action

$$S = \frac{R^2}{4\pi} \int_\Sigma \partial\phi \cdot \partial\phi$$



Equation of motion

$$\partial^2\phi = 0.$$

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$$\partial_i\phi = \sum_{j=1}^{2} \epsilon_{ij}\partial_j\sigma.$$

$$\epsilon_{ij} = -\epsilon_{ji}, \qquad \epsilon_{12} = 1.$$

Classical Duality

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$$\partial_i \phi = \epsilon_{ij}\, \partial_j \sigma,$$

$$\partial_i \sigma = -\epsilon_{ij}\, \partial_j \phi$$

Dual Picture

$$\partial^2 \sigma = 0.$$

Duality in QFT

$$\phi \,:\, \Sigma \to S^1$$

$$\frac{1}{4\pi}\int_\Sigma \partial\Phi^* \cdot \partial\Phi, \qquad \Phi = e^{i\,R\,\phi}$$

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Covariant Derivative

$$\mathcal{D}\Phi = (\partial + i\,R\,A)\Phi$$

$$D\phi = \partial\phi + A$$

Big theory

$$S = \frac{R^2}{4\pi}\int D\phi \cdot D\phi - \frac{i}{2\pi}\int \sigma\, F_A$$

Equations of Motion

$$\mathcal{L}_\sigma : F_A = 0, \quad (1)$$

$$\mathcal{L}_\phi : \partial^2\phi = 0, \quad (2)$$

$$\mathcal{L}_A : \partial_i\phi = \frac{i}{R^2}\epsilon_{ij}\partial_j\sigma. \quad (3)$$

Dual Theory

$$\phi = 0,$$

$$S = \frac{R^2}{4\pi}\int A\cdot A - \frac{i}{2\pi}\int C\cdot A, \qquad C_i = \epsilon_{ij}\partial_j\sigma$$

Dual Theory

$$\phi = 0,$$

$$S = \frac{R^2}{4\pi}\int A\cdot A - \frac{i}{2\pi}\int C\cdot A, \qquad C_i = \epsilon_{ij}\partial_j\sigma$$

Integrate Out A

$$A \;\rightarrow\; A' = A - \frac{i}{R^2}C$$

$$S = \frac{1}{4\pi\, R^2}\int (\partial\sigma)^2.$$

$R \to 1/R$

$$Z = \int \mathfrak{D}\phi \, e^{-S/\hbar}$$

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$$\hbar \to \; R^4/\hbar$$

Correlation functions

$$\left\langle D\phi(x) \prod_i \mathcal{O}_i(x_i) \right\rangle$$

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Dual Picture

$$D\phi \to \frac{i}{R^2}\epsilon_{ij}\partial_j\sigma$$

Correlation functions

$$\left\langle e^{i\phi(x)} e^{-i\phi(y)} \right\rangle$$

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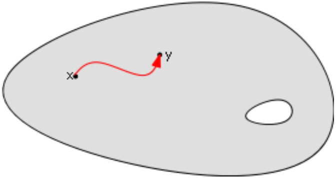
Wilson Line

$$\left\langle e^{i\phi(x)} e^{\int_x^y A} e^{-i\phi(y)} \right\rangle$$

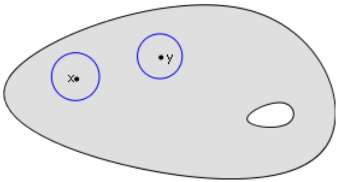


Dual Picture

$$\Sigma \to \Sigma - \{x, y\}$$

Winding Numbers

$$\oint_x d\sigma = 1, \qquad \oint_y d\sigma = -1.$$

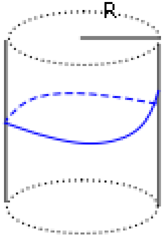


1. $\hbar \leftrightarrow 1/\hbar$ implies that a classical theory can be dual to a quantum theory,
2. A local operator can be dual to a non-local procedure.

Reference

E. Witten, *Dynamics of Quantum Field Theory*, in: P. Deligne et al., eds., *Quantum fields and Strings: A Course for Mathematicians*, AMS 1999.

Closed String



Mass formula

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\ell_s^4} + \frac{2}{\ell_s^2}(N_l + N_r - 2) \quad (4)$$

$$0 = n\,w + N_l - N_r \quad (5)$$

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T-duality

$$R \to R' = \frac{\ell_s^2}{R}, \qquad n \leftrightarrow w$$

$$0 < R < \ell_s \qquad R > \ell_s$$

Gauge Symmetry

$$m^2 = 0 \Rightarrow n = w = 0, \qquad N_l = N_r = 1.$$

Generic gauge group

$$U(1)_l \times U(1)_r$$

Self-dual Radius $R = \ell_s$

New massless states

$$n = w = \pm 1,\ N_l = 0,\ N_r = 1, \quad n = -w = \pm 1,\ N_l = 1,\ N_r = 0,$$
$$n = \pm 2,\ w = N_l = N_r = 0, \quad w = \pm 2,\ n = N_l = N_r = 0.$$

Enhanced gauge symmetry

$$SU(2)_l \times SU(2)_r$$

1. $R \to 0$ and $R \to \infty$ are physically equivalent.
2. Gauge symmetries can be accidental.

Reference

J. Polchinski, *String theory: An introduction to the Bosonic String*, Cambridge University Press, 1998.

Classical Virasoro algebra

Conformal transformation on $\mathbb{R}^2 \cup \infty$

$$ds^2 = dx^2 + dy^2$$

$$z = x + i\,y \qquad ds^2 = dz\,d\bar{z}$$

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$$\begin{aligned} z &\rightarrow z' = f(z) \\ ds^2 &\rightarrow |\partial f|^2\,dz\,d\bar{z} \end{aligned}$$

Generators

$$\ell_n = -z^{n+1}\partial_z,$$

Witt Algebra

$$[\ell_n, \ell_m]_{\text{Lie}} = (n-m)\,\ell_{n+m}$$

Noether Charges

$$T_{\mu\nu} = -\frac{4\pi}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$\partial_\mu T^{\mu\nu} = 0, \qquad T^\mu_\mu = 0$$

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$$T_{zz} = T(z), \quad T_{\bar{z}\,\bar{z}} = \bar{T}(\bar{z}), \quad T_{z\,\bar{z}} = 0$$

Virasoro Algebra

$$L_n = \frac{1}{2\,\pi\,i} \oint dz\, z^{n+1}\, T(z)$$

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Central Charge

$$\langle T(z)\, T(w) \rangle = \frac{c/2}{(z-w)^4}.$$

Geometry

$\mathbb{R}^{(2,2)}$

$$ds^2 = dX_0^2 + dX_1^2 - dX_2^2 - dX_3^2$$

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AdS_3 Geometry

$$X_0^2 + X_1^2 - \left(X_2^2 + X_3^2\right) = \ell^2$$

$$ds^2 = r^2\left(-dt^2 + d\phi^2\right) + \frac{dr^2}{r^2}$$

Asymptotic Symmetries

$$ds^2 = r^2\left(-dt^2 + d\phi^2\right) + \frac{dr^2}{r^2}$$

Diffeomorphisms

$$\delta\, g_{\mu\nu} = \nabla_\mu\, \xi_\nu + \nabla_\nu\, \xi_\mu$$

$$\begin{array}{lll} \delta g_{tt} = \mathcal{O}(1) & \delta g_{\phi\phi} = \mathcal{O}(1) & \delta g_{rr} = \mathcal{O}(r^{-3}) \\ \delta g_{t\phi} = \mathcal{O}(1) & \delta g_{t\,r} = \mathcal{O}(r^{-1}) & \delta g_{\phi\, r} = \mathcal{O}(r^{-1}) \end{array}$$

Generators of asymptotic symmetry

$$\xi = \xi(z)\partial_z - \frac{1}{2}(\partial_z \xi) r\, \partial_r + c.c.$$

$$z = \ell\, \phi + i\, t_E$$

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$$\xi = \xi(z)\partial_z - \frac{1}{2}(\partial_z \xi) r\, \partial_r + c.c.$$

$$z = \ell\,\phi + i\, t_E$$

central charge

$$c = \frac{3\ell}{2G} \gg 1$$

J.D. Brown, M. Henneaux, Commun. Math. Phys. 104, (1986) 207.

Entropy

Modular invariance

$$Z(\beta) = Z\left(\frac{4\pi^2}{\beta}\right)$$

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Modular invariance

$$Z(\beta) = Z\left(\frac{4\,\pi^2}{\beta}\right)$$

$$Z(\beta) \simeq \exp\left(\frac{4\,\pi^2\,c}{12\beta}\right)$$

Cardy formula

$$S = \frac{2\,\pi^2\,c\,T}{3}$$

J.L. Cardy, Nucl. Phys. B. 270, (1986) 186.

The number of spacetime dimensions might be physically unimportant!

Suggested Reading

J. Polchinski, *Dualities*, arXiv: 1412.5704