General Relativity vs. Quantum mechanics

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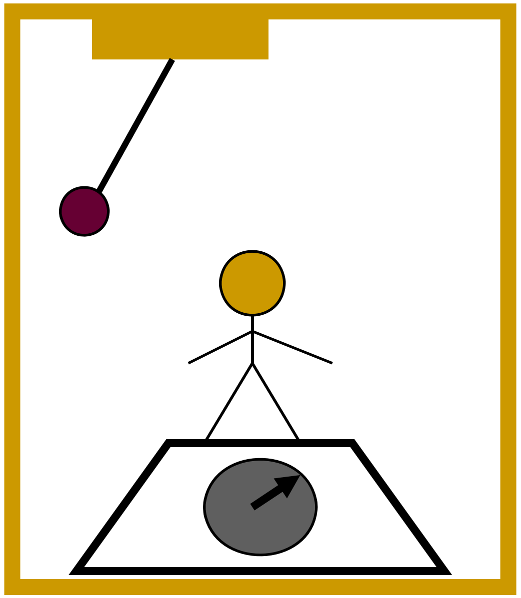
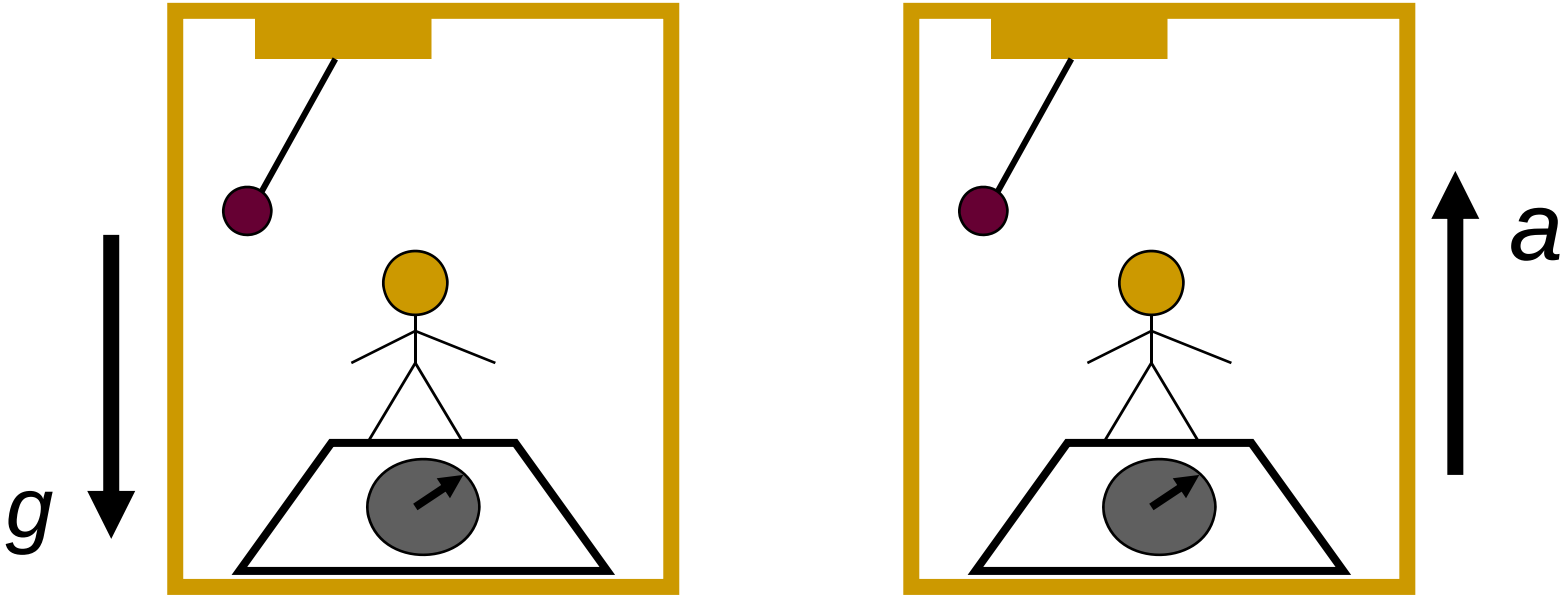
The equivalence principle

Imagine a scientist who is measuring the period of a pendulum or his own weight.



The equivalence principle



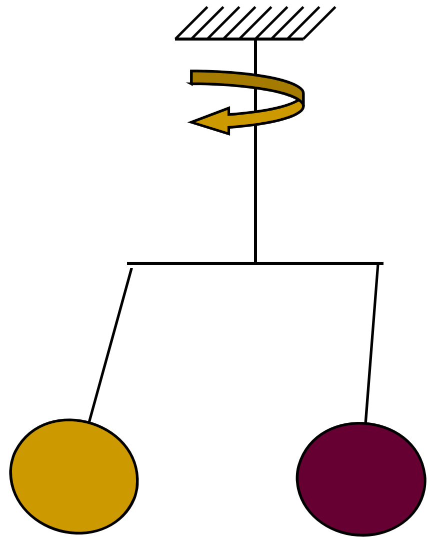
Can he recognize whether he is experiencing a gravitational field *g* or an acceleration $a=g$?

The equivalence principle

$$F = \frac{GMm^*}{r^2} = ma \quad \Rightarrow \quad a = \left(\frac{GM}{r^2}\right)\frac{m^*}{m}$$

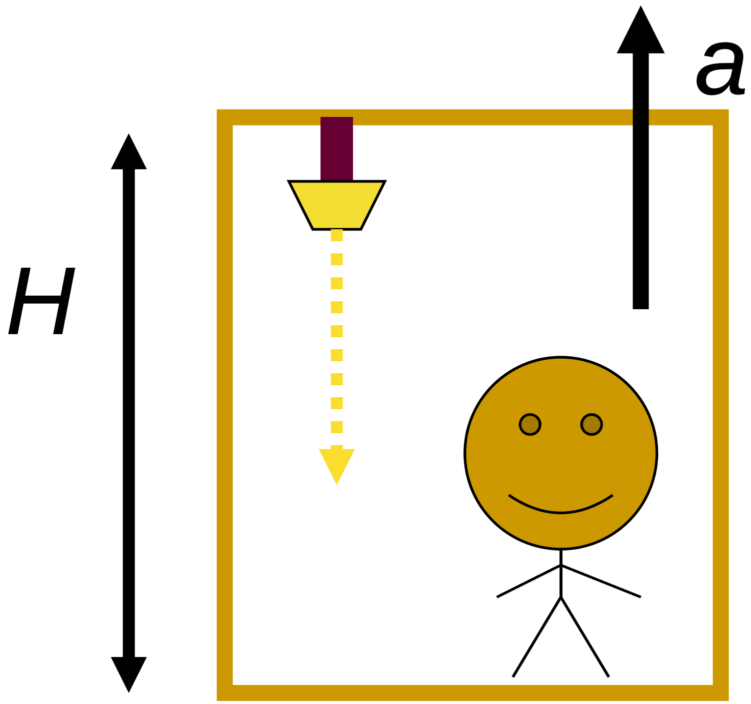
The equivalence principle

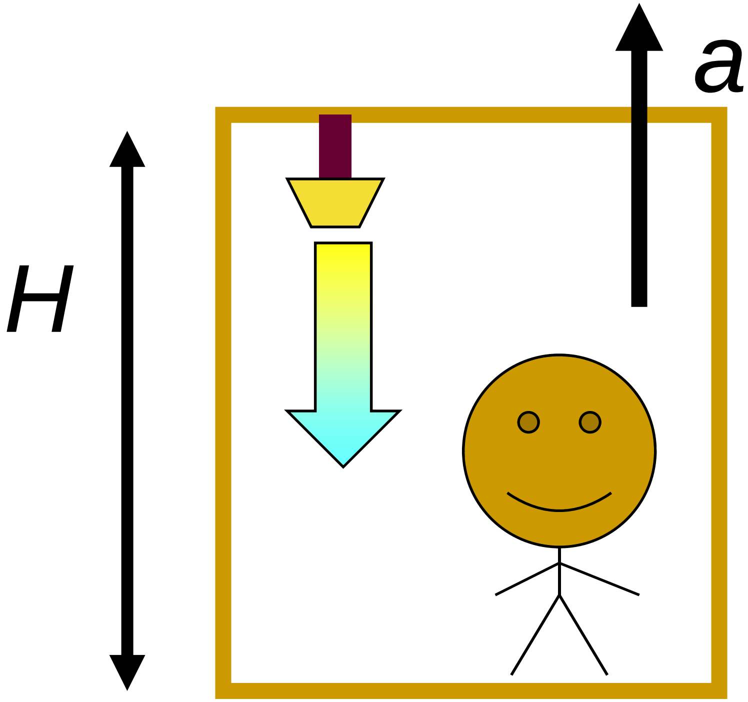
$$a = \left(\frac{GM}{r^2} \right) \frac{m^*}{m}$$



Gravitational blue-shift

A spacecraft with a sodium lamp on the ceiling.



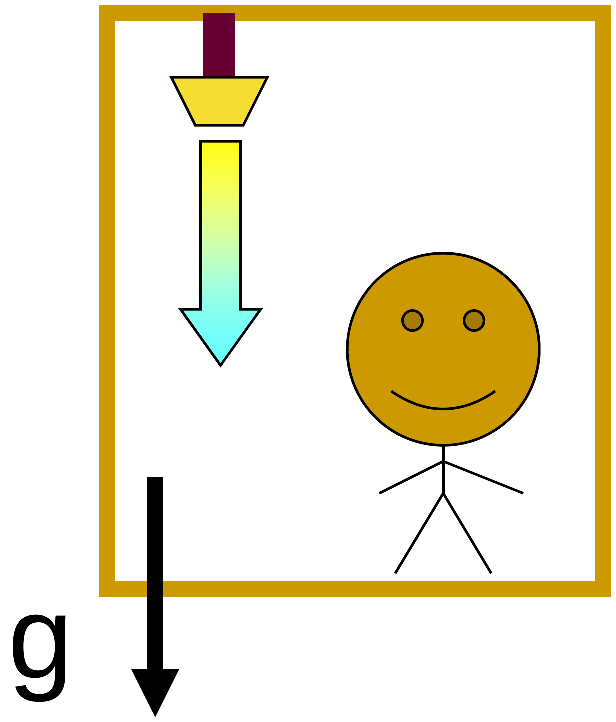


$$\Delta u = at$$

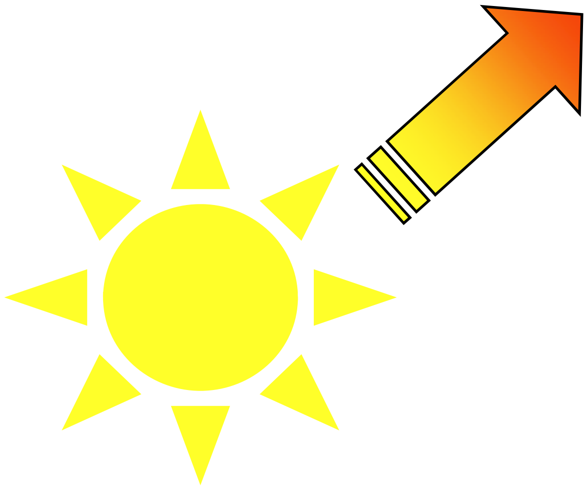
$$\nu' = \nu\sqrt{\frac{1+\Delta u/c}{1-\Delta u/c}} \approx \nu\left(1+\Delta u/c\right)$$

$$\frac{\nu' - \nu}{\nu} = \frac{aH}{c^2}$$

The equivalence principle implies that there is a gravitational blue shift.



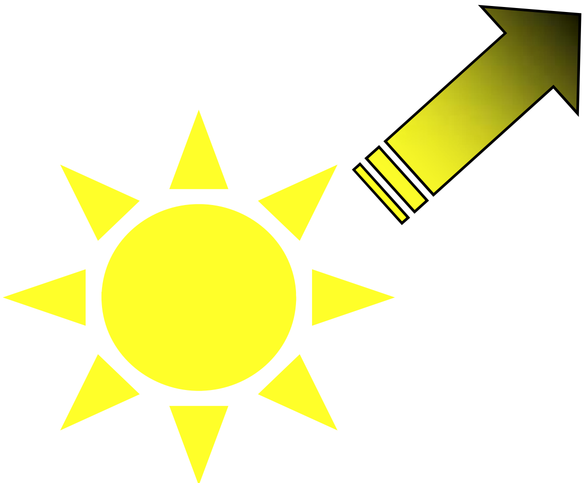
Gravitational red shift



$$\frac{\Delta \nu}{\nu} = -\frac{GM}{Rc^2}$$

Black hole

$$f' = \sqrt{1 - \frac{2GM}{R}}\, f$$

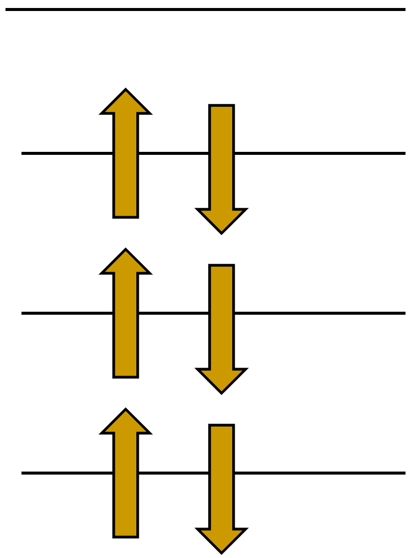


White dwarf

$$E = \frac{3}{5} N_e E_F - \frac{3}{5} \frac{GM^2}{R}$$

$$E = E_F$$

$$E = 0$$



White dwarf

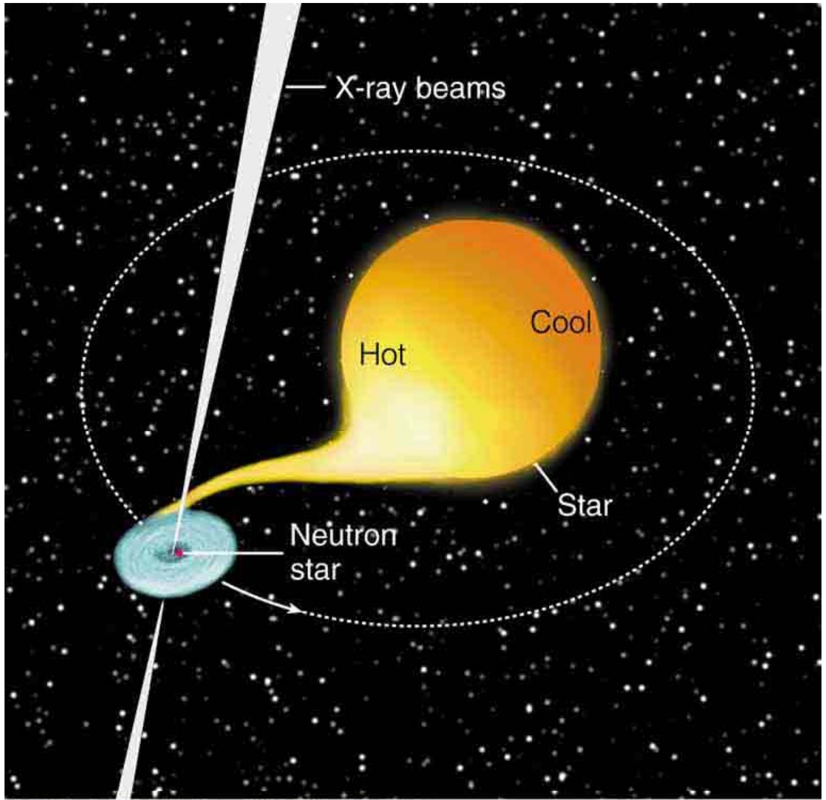
$$E_F = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 N_e}{V}\right)^{2/3}$$

$$R \propto \frac{N_e^{5/3}}{N_n^2}$$

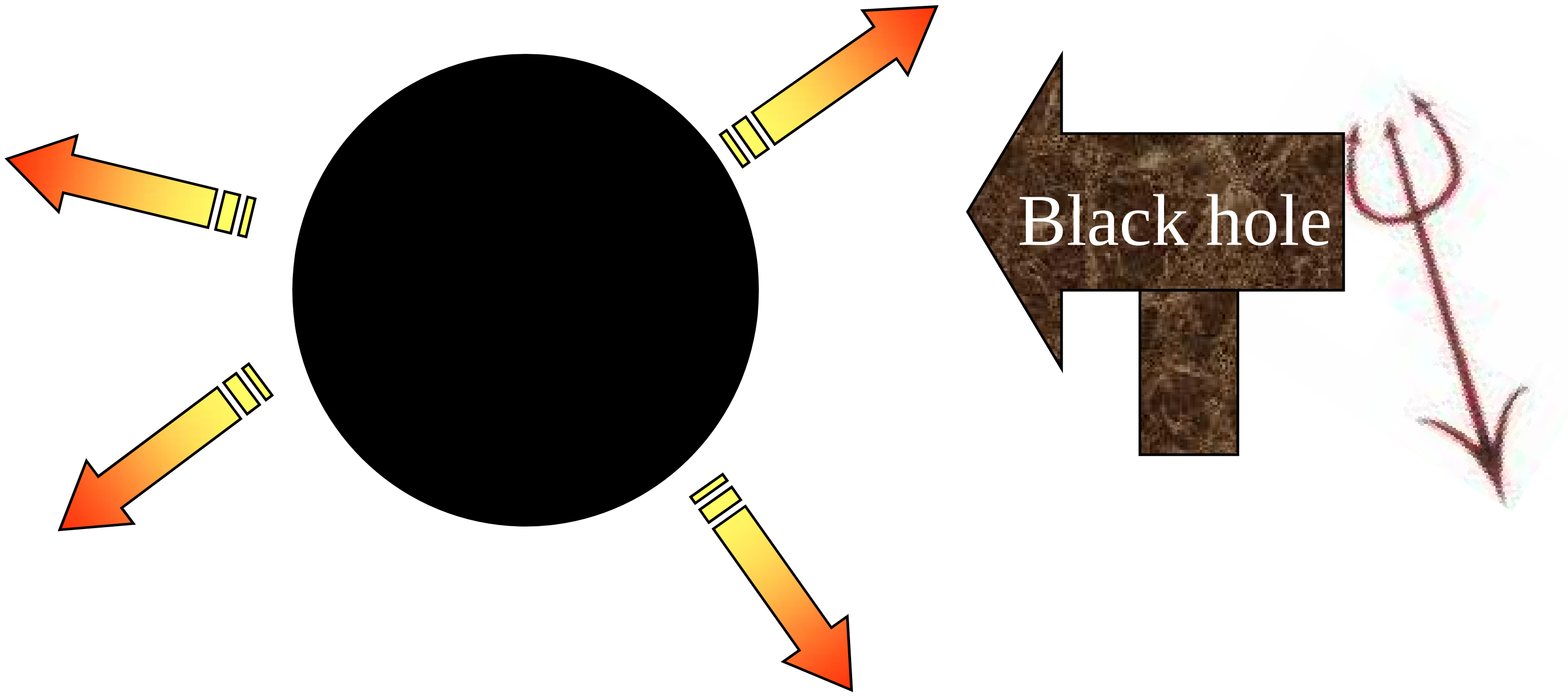
$$E_F \propto N_e^{-8/3}$$

$$e + p \rightarrow n + \nu_e$$

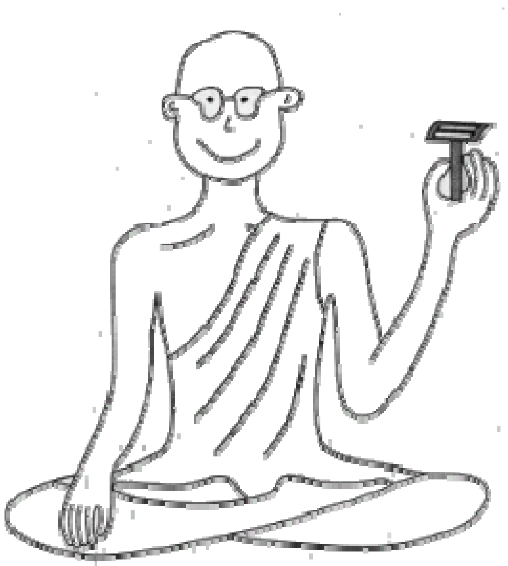
Neutron star



- The radius of a neutron star of solar mass is approximately 10Km.


Black hole

No-hair theorem

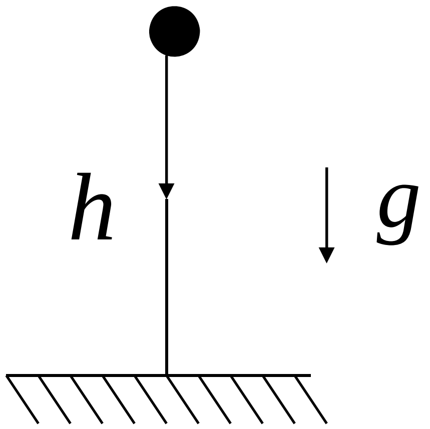


A black hole is uniquely determined by its mass, electrical charge and angular momentum.

The Bekenstein-Hawking entropy

$$S = \frac{c^3 k_B}{4G\hbar} A$$

Dimensional analysis



$$T = (h/g)^{1/2}$$

Area of the event horizon

$$A = \frac{m^2 G^2}{c^4}$$

Entropy of the black hole

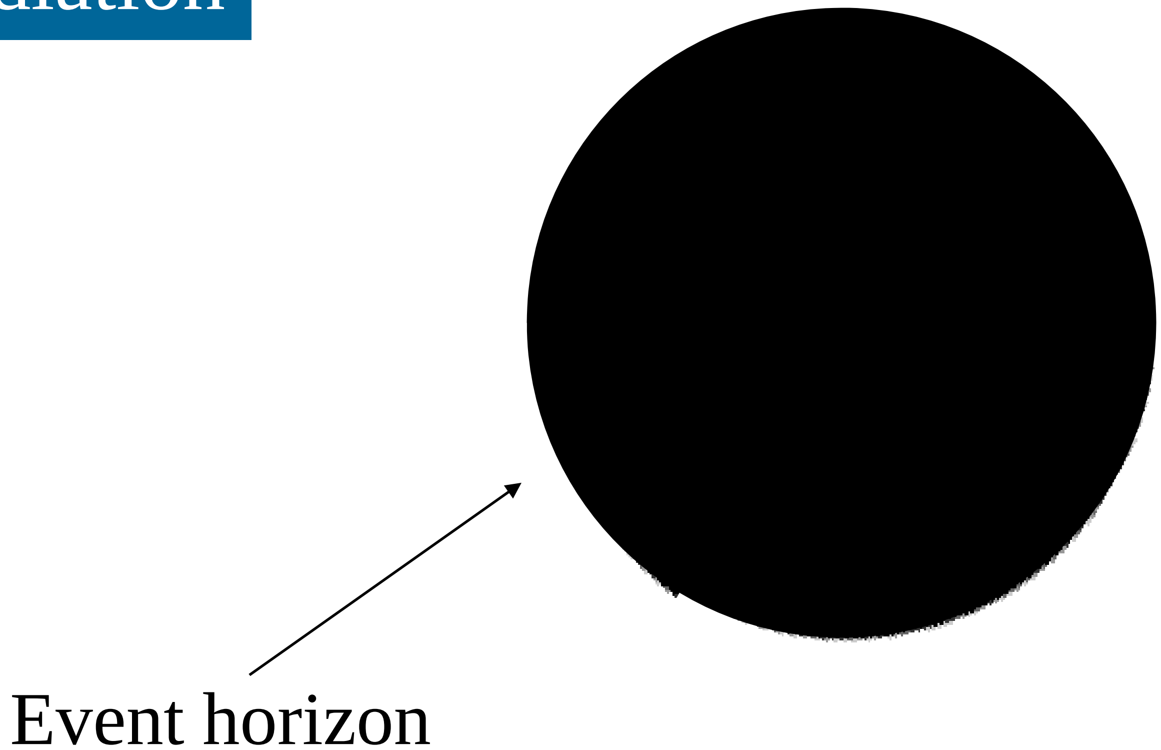
$$S = \frac{c^3 k_B}{G h} A$$

Hawking temperature

$$\theta_H = \frac{c^3 h}{G k_B} \frac{1}{m}$$

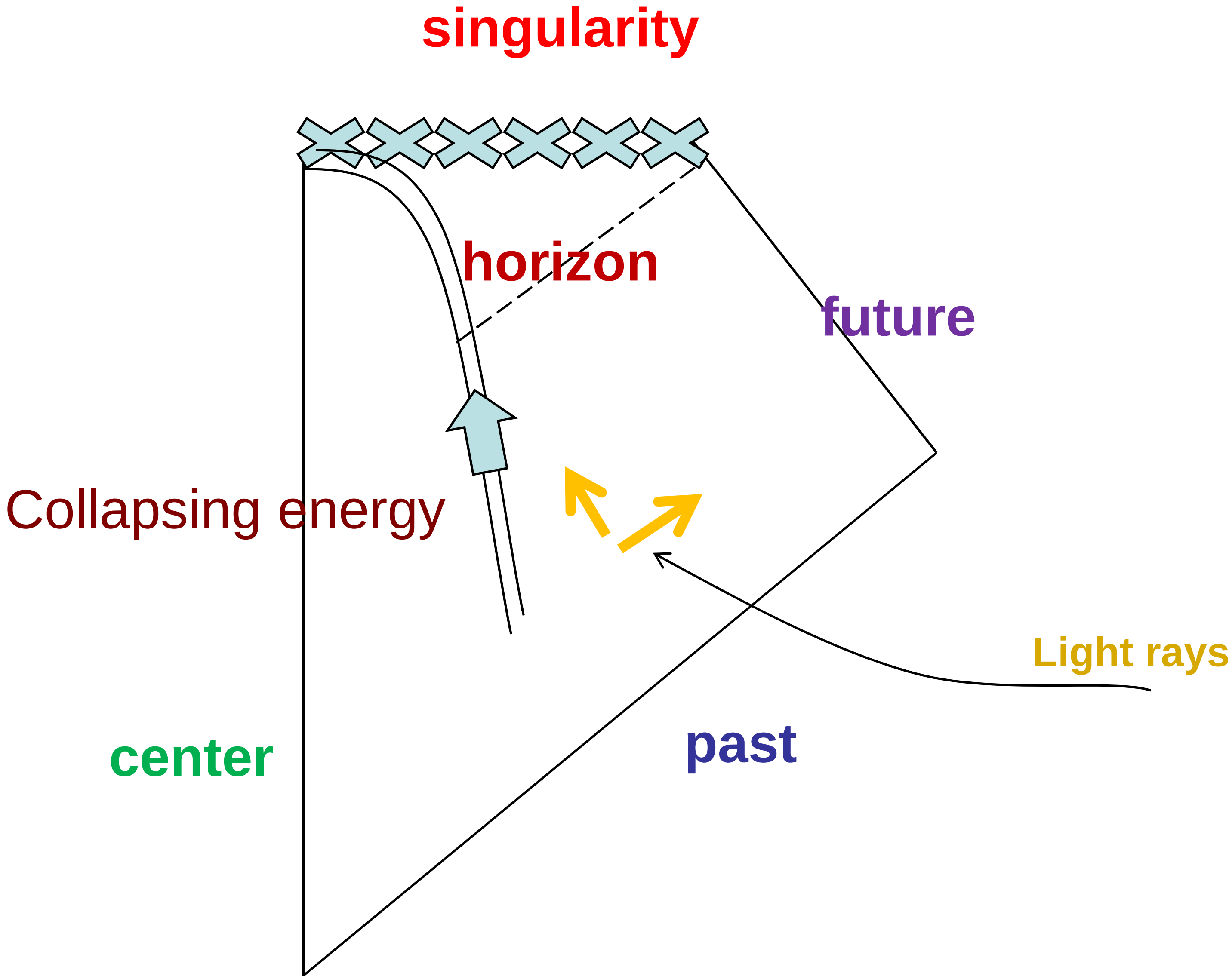
$$d\left(mc^2\right) = \theta_H \, dS$$

Hawking radiation



Evaporation time

$$t^{*} = \frac{G^{2}}{3c^{4}h} m^{3} \approx 10^{71} \left(\frac{m}{M_{*}} \right)^{3} s$$


singularity
horizon
future
Collapsing energy
Light rays
center
past

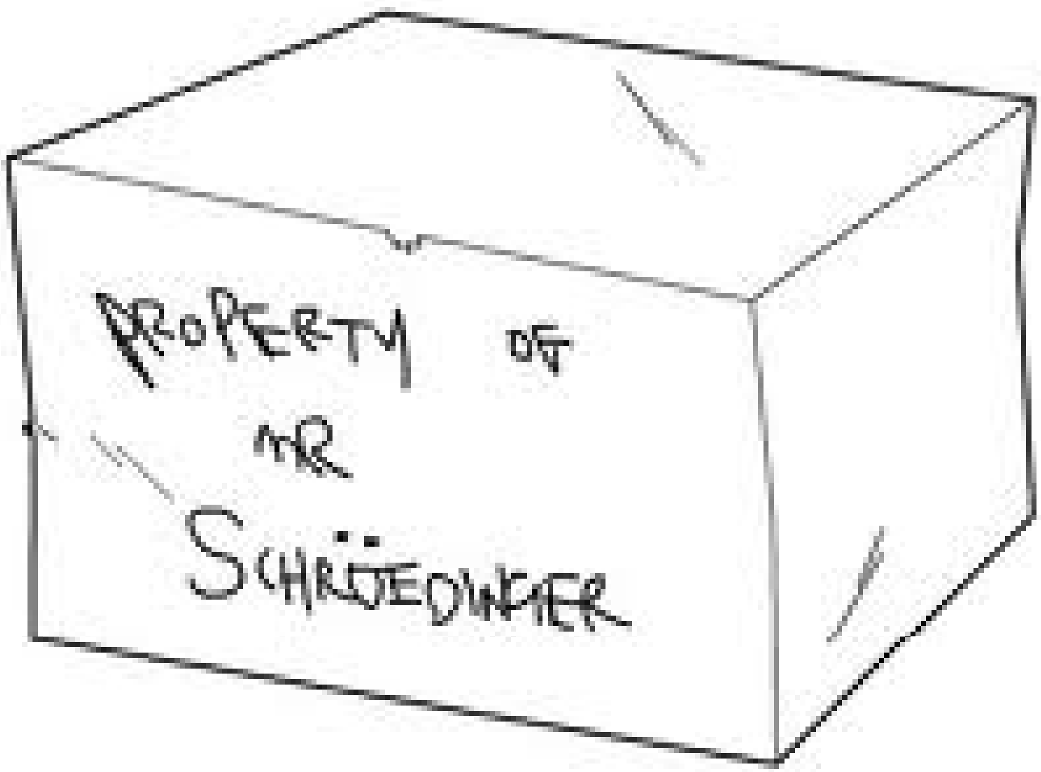
$$\left|\psi_{\text{out}}\right\rangle = S\left|\psi_{\text{in}}\right\rangle$$

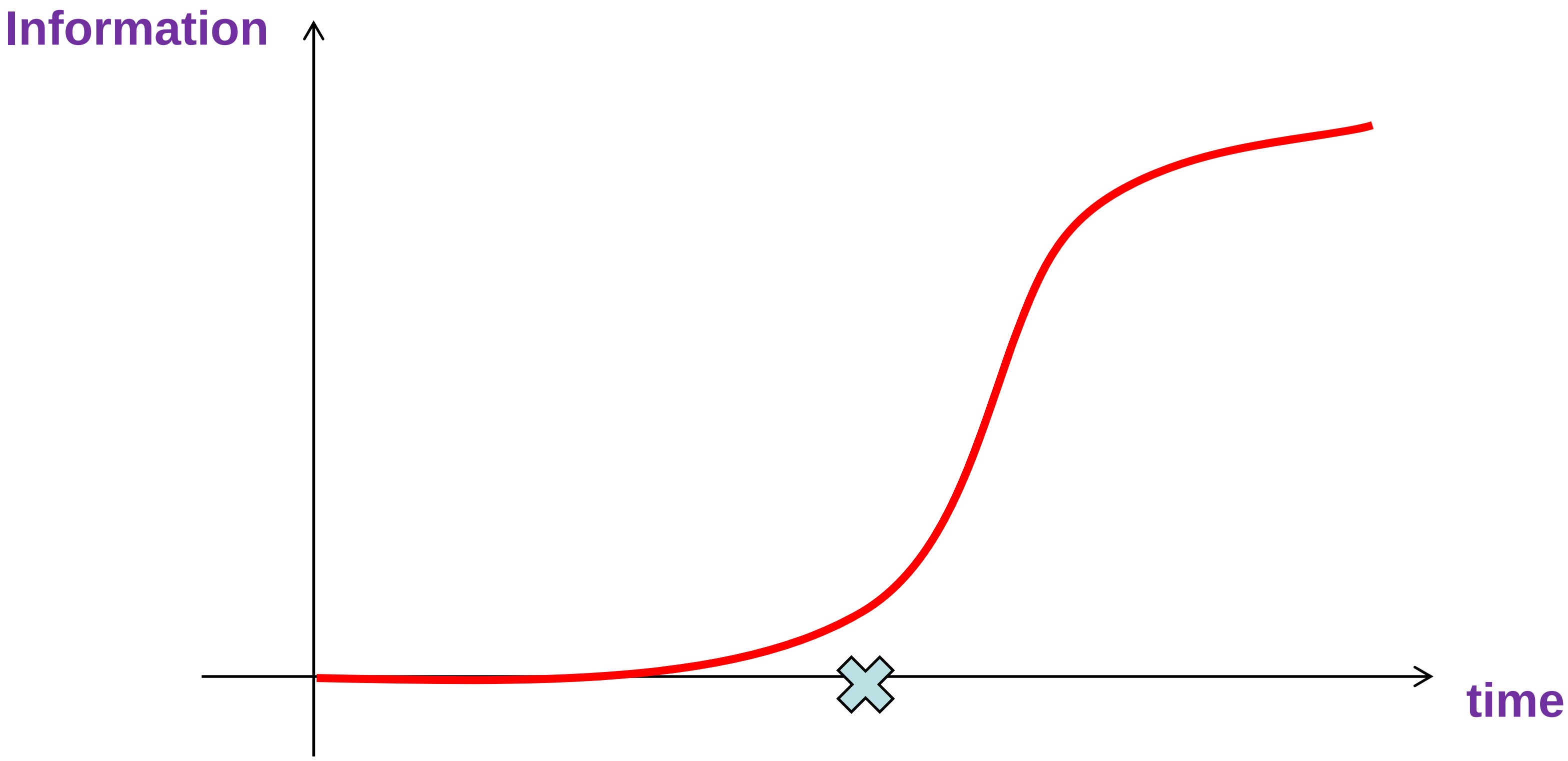
$$\left|\psi_{\text{in}}\right\rangle = S^{+}\left|\psi_{\text{out}}\right\rangle$$

$$H_{\text{out}} = H_{\text{future}} \otimes H_{\text{singularity}}$$

$$\rho_{\text{out}} = \text{Tr}_{\text{singularity}}\left|\psi_{out}\right\rangle\left\langle\psi_{out}\right|$$

Bomb in the box





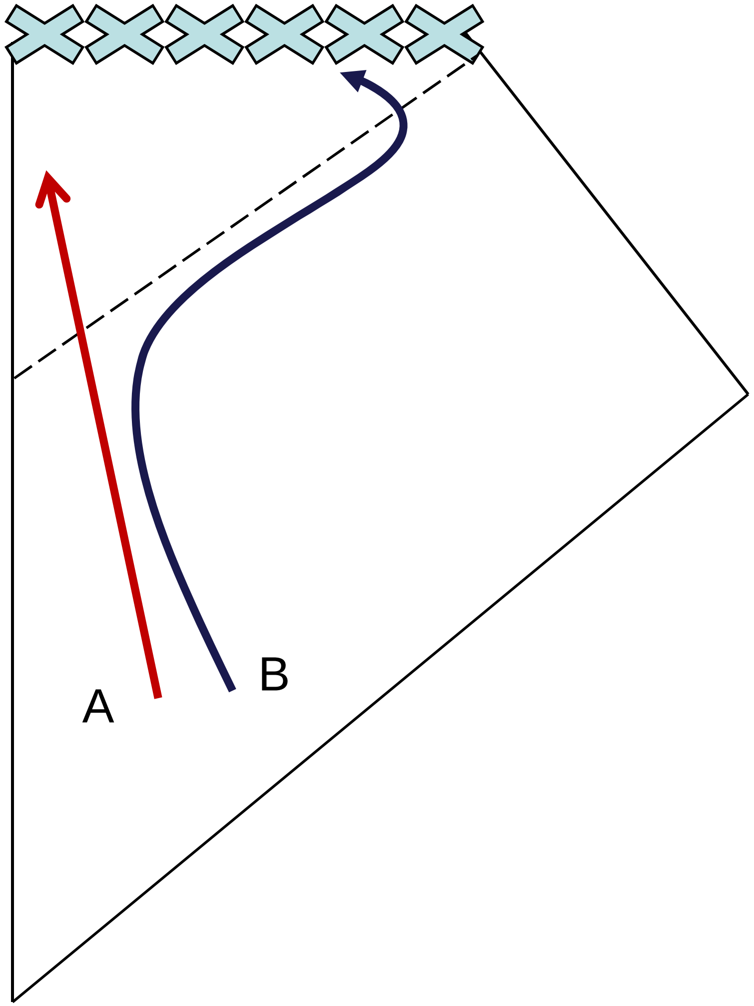
Don N. Page, “Information in Black hole radiation”, PRL 71 (1993) 3743.

Black holes as mirrors

k qubits dumped into an old black hole will be revealed after just few more than *k* qubits are emitted in the Hawking radiation.

P. Hayden, J. Preskill, arXiv:0708.4025

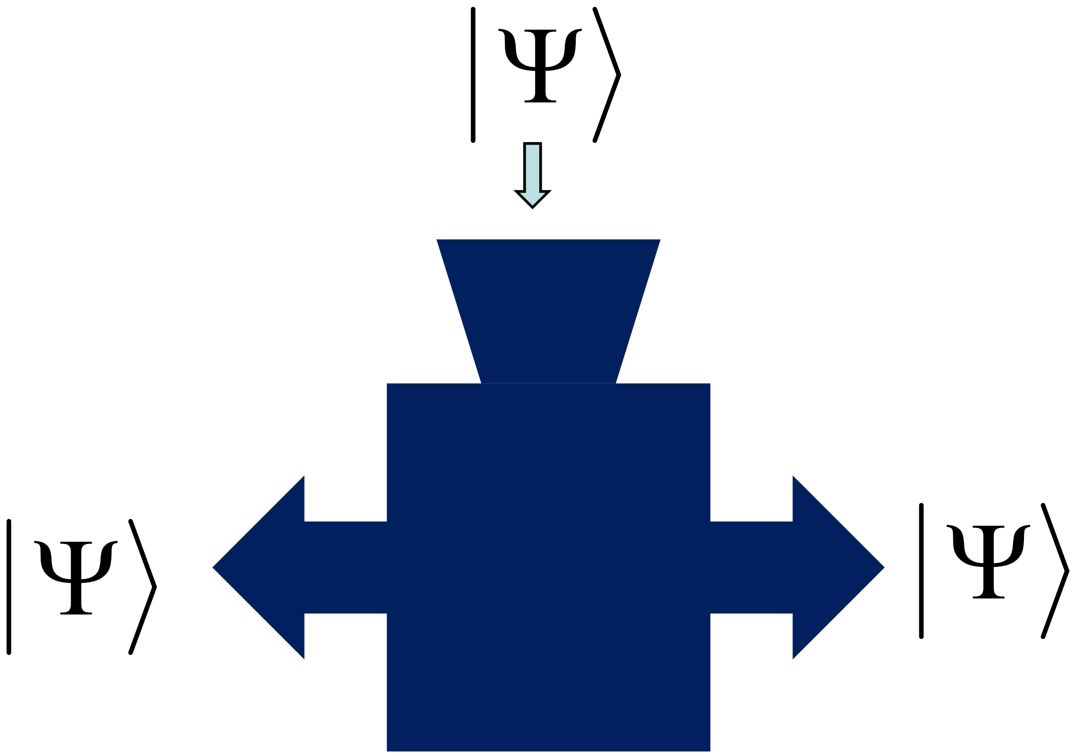
Black hole complementarity



Leonard Susskind, James Lindesay, An Introduction To Black Holes, Information And The String Theory Revolution: The Holographic Universe

The quantum Xerox principle

(the no-cloning principle)



Leonard Susskind, James Lindesay, An Introduction To Black Holes, Information And The String Theory Revolution: The Holographic Universe

$$|\uparrow\rangle \rightarrow |\uparrow\rangle|\uparrow\rangle$$

$$|\downarrow\rangle \rightarrow |\downarrow\rangle|\downarrow\rangle$$

$$\left(|\downarrow\rangle + |\uparrow\rangle\right) \overset{?}{\rightarrow} \begin{cases} |\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle \\ \left(|\downarrow\rangle + |\uparrow\rangle\right)\left(|\downarrow\rangle + |\uparrow\rangle\right) \end{cases}$$